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SOME PROPERTIES OF FUZZY FILTERS IN BCI/BCK-ALGEBRAS

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Abstract

BCI, BCK, MV-algebras arose as the algebras of non classical logic in the same way as boolean algebra arose as the algebra of classical logic. As a dual to the notion of fuzzy ideals of BCI-algebra, in [4], we have introduced the notion of fuzzy filters and established their basic properties and characterization. In this paper, we consider the notion of ultrafilters and show that such fuzzy filters take only the values {0, 1} and have level filters which are maximal filters. It is shown that fuzzy ultrafilters of MV-algebra are fuzzy primes and that fuzzy ideals and fuzzy filters come in pairs. Finally, we established some algorithms for filters and fuzzy filters.

1 Backgrounds

A BCI algebra is a non empty set X with a binary operation $*$ and a constant 0 satisfying the following axioms:

(1) $[(x*y)*(x*z)]*(z*y)=0$
(2) $[x*(x*y)]*y)=0$
(3) $x*x=0$
(4) $x*y=0 \; and \; y*x=0 \Longrightarrow x=y$

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(5) $x * 0 = 0 \Longrightarrow x = 0$

A partial ordering $\leq$ on X can be defined by $x \leq y$ if and only if $x * y = 0$. Further, if $x \geq 0 \; \forall \; x \in X$, then X is called a BCK-algebra. If a BCK-algebra satisfies the identity

$$x * (x * y) = y * (y * x),$$

then it is called commutative, in this case $x * (x * y) = y * (y * x)$ is the greatest lower bound $x \wedge y$ of x and y. If a commutative BCK-algebra has an upper bound 1, then the least upper bound $x \vee y$ of two elements x and y is given by $x \vee y = 1 * [(1 * x) \wedge (1 * y)]$. This gives the algebra the structure of bounded distributive lattice. We shall regard an MV-algebra as a bounded commutative BCK-algebra. The usual MV-algebra operations are given by

$x' = 1 * x$,

$xy = x * y'$ and

$x + y = (x'y')' = 1 * [(1 * x) * y]$.

Definition 1.1. An ideal of a BCI-algebra is a subset I containing 0 such that if $x * y \in I$ and $y \in I$, then $x \in I$.
If the algebra is commutative, then an ideal I is prime if it is proper and if whenever $x \wedge y \in I$, then $x \in I$ or $y \in I$.
An ideal I is maximal if it is proper and whenever $I \subset J$ for some ideal J, then $I = J$ or J is the whole algebra.

It is clear that the ideals of an MV-algebra are precisely the ideals of the underlying BCI-algebra.

Definition 1.2. A non empty set F of a BCI-algebra X is said to be a filter if

1. $x \in F$ *and* $x \leq y \Rightarrow y \in F$
2. $x \in F$ *and* $y \in F \Rightarrow x \wedge y \in F$ *and* $y \wedge x \in F$.

A filter F is prime if it is proper and if whenever $x \vee y \in F$, then $x \in F$ or $y \in F$.
A filter F is maximal if it is proper and whenever $F \subset U$ for some filter U, then $F = U$ or U is the whole algebra.

It is also clear that the filters of an MV-algebra are precisely the filters of the underlying BCI-algebra. We briefly review some fuzzy logic concepts, we refer the reader to [1], [3], [6], [7] for more details.

Definition 1.3. A fuzzy subset of a BCI-algebra X is a function

$$\mu : X \longmapsto [0, 1];$$

It is a fuzzy ideal if it satisfies $\mu(0) \geq \mu(x)$ and $\mu(x) \geq (\mu(x * y), \mu(y))$.
A fuzzy ideal μ is prime if $\mu(x \wedge y) = \max(\mu(x), \mu(y))$. We can define a partial ordering relation $\leq$ on a set of all fuzzy ideals of X by $\mu \leq \lambda$ if and only if $\mu(x) \leq \lambda(x) \; \forall \; x \in X$.
A fuzzy ideal is maximal if it is a maximal element of the set of all fuzzy ideals of X.

2 Fuzzy filters in BCI-algebra

Definition 2.1. *[4]*A fuzzy subset μ of a commutative BCI-algebra X is a fuzzy filter if it satisfies

$$\mu(x \wedge y) \geq \min(\mu(x), \mu(y))$$

and when $y \geq x$, we have

$$\mu(y) \geq \mu(x) \ \forall \ x \text{ and } y \in X.$$

We can characterize a fuzzy filter in a commutative BCI-algebras by the following proposition.

Proposition 2.1. *A fuzzy subset μ of a commutative BCI-algebras X is a fuzzy filter if and only if*

$$\mu(x \wedge y) = \min(\mu(x), \mu(y)) \forall \ x \text{ and } y \in X.$$

Sketch of Proof. For any x and y in X, we have $x \geq x \wedge y$ and $y \geq x \wedge y$. Using the definition of fuzzy filters, we obtain $\mu(x \wedge y) = \min(\mu(x), \mu(y))$. □

Example 2.1. Every constant function $\mu : X \longmapsto [0, 1]$ is a fuzzy filter.

Example 2.2. Let $X = \{0, 1, 2, 3\}$ with $*$ defined by the following table:

$*$	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	1	0	0
3	3	2	1	0

It is easy to check that X is a commutative BCI-algebra. Let μ to be a fuzzy subset on X defined by $\mu(1) = \mu(3) = \mu(2) > \mu(0) = \mu(1)$, routine calculations prove that μ is a fuzzy filter.

Example 2.3. Let X ={0, a, b, c, 1} with $*$ defined by the following table:

$*$	0	a	b	c	1
0	0	0	0	0	0
a	a	a	0	0	0
b	b	a	0	a	0
c	c	c	c	0	0
1	1	c	c	a	0

Let μ to be a fuzzy subset on X defined by $\mu(a) = \mu(b) = \mu(0) < \mu(c) = \mu(1)$, one can easily check that μ is a fuzzy filter.

Given a fuzzy subset μ and $t \in [0,1]$, $\mu_t = \{x \in X/\mu(x) \geq t\}$. This could be an empty set. The Theorem 2.2. of [4] shows that μ is a fuzzy filter if and only if μ_t is either empty or a filter. Thus, given a fuzzy filter of X, $X_\mu = \{x \in X/\mu(x) = \mu(1)\}$ is a filter.
If F is a filter, then the characteristic function of F, χ_F is a fuzzy filter. Clearly, given a filter F of X,

$$X_{\chi F} = \{x \in X/\chi_F(x) = 1\} = F$$

3 Fuzzy prime filter in MV-algebra

In this section, X will always denote a bounded commutative BCK-algebra.

Definition 3.1. A fuzzy filter μ of an MV-algebra X is prime if it is non constant and

$$\mu(x \vee y) = \max(\mu(x), \mu(y)) \ \forall \ x \ and \ y \in X.$$

It is shown (Theorem 2.1. of [4]) that F is a filter of X if and only if the characteristic function of F is a fuzzy filter. In a similar way, we can show the following result.

Theorem 3.1. *A filter F of an MV-algebra X is prime if and only if the characteristic function of F, χ_F is a fuzzy prime filter.*

We can characterize fuzzy prime filter in terms of level subsets as:

Theorem 3.2. *A fuzzy filter μ of an MV-algebra X is prime if and only if*

$$\mu_t = \{x \in X/\mu(x) \geq t\}$$

is either empty or a prime filter of X.

Proof. Suppose that μ is a fuzzy prime filter, we already know (Theorem 2.2. of [4]) that μ_t is a fuzzy filter. Next, let $x \vee y \in \mu_t$. Then $\mu(x \vee y) \geq t$. Since μ is fuzzy prime,

$$\mu(x \vee y) = \max(\mu(x), \mu(y)).$$

So $\mu(x) \geq t$ or $\mu(y) \geq t$. We have $x \in \mu_t$ or $y \in \mu_t$, which prove that μ_t is prime.

Conversely, suppose that $\mu_t = \{x \in X/\mu(x) \geq t\}$ is a prime filter of X. According to Theorem 2.2. of [4], μ_t is a fuzzy filter. Let x and $y \in X$ and $t = \mu(x \vee y)$, $x \vee y \in \mu_t$. Since μ_t is a prime filter, we have $x \in \mu_t$ or $y \in \mu_t$. So $\mu(x) \geq t$ or $\mu(y) \geq t$ and we obtain that

$$\max(\mu(x), \mu(x)) \geq t = \mu(x \vee y).$$

On the other hand $x \leq x \vee y$ and $y \leq x \vee y$, we apply Definition 2.1 and obtain $\mu(x) \leq \mu(x \vee y)$ and $\mu(y) \leq \mu(x \vee y)$ so that $\max(\mu(x), \mu(x)) \leq \mu(x \vee y)$. Finally $\max(\mu(x), \mu(x)) = \mu(x \vee y)$ and we conclude that μ is a fuzzy prime filter. □

Now, we construct a new fuzzy prime filter from a given prime filter.

Definition 3.2. *[4]*Let μ is a fuzzy subset of X and $\alpha \in [0, 1]$. The function

$$\mu^{\alpha} : X \longmapsto [0, 1]$$

is given by $\mu^{\alpha}(x) = (\mu(x))^{\alpha}$.

Proposition 3.1. *If a fuzzy subset μ of X is a fuzzy prime filter, then μ^{α} is also a fuzzy prime filter.*

Sketch of Proof. From Theorem 2.3. of [4], we have that μ^{α} is also a fuzzy filter when μ is a fuzzy filter. Combining the definition of fuzzy prime filter and the definition of μ^{α}, we can easily obtain the result. □

Definition 3.3. *[4]*Let $f : X \longmapsto Y$ be a mapping and μ a fuzzy subset of $f(X)$. Then $f^{-1}(\mu)(x) = \mu(f(x))$ is a fuzzy subset. Conversely, let λ be a fuzzy subset of X. Then $f(\lambda)$ is defined by:

$$f(\lambda(x)) = \sup_{t \in f^{-1}(y)} \lambda(t)$$

is a fuzzy subset of Y.
A mapping f is called an MV-homomorphism if

$$f(x * y) = f(x) * f(y).$$

It is clear that for any MV-homomorphism f, we have $f(0) = 0$ and $f(x) \leq f(y)$ when $x \leq y$.

Proposition 3.2. *Let $f : X \longmapsto Y$ be an onto MV-homomorphism, we have the following results:*

- *If μ is a fuzzy prime filter, then $f^{-1}(\mu)$ is also a fuzzy prime filter.*
- *Conversely, if λ is a fuzzy prime filter with a* sup *property (for any subset T of X, there exists $t_0 \in T$ such that $\lambda(t_0) = \sup_{t \in T} \lambda(t)$) is also a fuzzy prime filter.*

The proof is similar to the one of Theorem 2.5. of [4] and is omitted.

4 Fuzzy ultrafilter in a bounded commutative BCK-algebra

In this section, X will always denote a bounded commutative BCK-algebra. If μ is a fuzzy filter of X, it is easy to prove that $\mu(1)$ is the largest value of μ. It is often convenient to have $\mu(1) = 1$. A fuzzy filter μ is normalized if $\mu(1) = 1$. The normalization of μ is

$$\mu^+ : X \longmapsto [0,1]$$
$$x \longrightarrow \mu^+(x) = \mu(x) + 1 - \mu(1).$$

It is easy to prove that μ^+ is a normalized fuzzy filter and $\mu^+ = \mu$ if μ is normalized. We can define a partial ordering on the set of all fuzzy filters of X by $\mu_1 \leq \mu_2$ if $\mu_1(x) \leq \mu_2(x)\ \forall\ x \in X$. It is easy to see that $\mu \leq \mu^+$. Let $\Im(X)$ denote the set of all normalized fuzzy filters μ of X such that $0 \in$ image of μ. The restriction of $\leq$ to $\Im(X)$ is a partial order. If $\mu_1 \leq \mu_2$, we have $X_{\mu_1} \leq X_{\mu_2}$. It is easy to see that for any proper filter F of X, $\chi_F \in \Im(x)$ and for two proper filters F_1, F_2 of X, we have

$$F_1 \subset F_2 \Longleftrightarrow \chi_{F_1} \leq \chi_{F_2}.$$

We can establish a correspondence between filters and fuzzy filters of X as follows:

Let $\Im(X)$ be the set of normalized fuzzy filters of X and $F(X)$ the set of proper filters of X. We define θ and γ in the following way,

$$\theta : F(X) \longmapsto \Im(X)$$
such that $\theta(F) = \chi_F$
$$\gamma\colon \Im(X) \longmapsto F(X)$$
such that $\gamma(\mu) = X_\mu$

Proposition 4.1. *θ is injective and γ is surjective.*

Sketch of Proof. We can easily establish that for any filter F of X, $\gamma(\theta(F)) = F$ and for any fuzzy filter μ of $\Im(X)$, $\theta(\gamma(\mu)) = \chi_{X_\mu} \leq \mu$. □

Definition 4.1. Let μ_1 and μ_2 two fuzzy subsets of an MV-algebra X, we define $\mu_1 \wedge \mu_2$: $X \longmapsto [0,1]$ by $(\mu_1 \wedge \mu_2)(x) = \mu_1(x) \wedge \mu_2(x)$.

One can easily establish the following lemmas:

Lemma 4.1. *If μ_1 and μ_2 are two fuzzy filters of an MV-algebra X, then $\mu_1 \wedge \mu_2$ is also a fuzzy filter. Furthermore, if μ_1 and μ_2 are normalized, then $\mu_1 \wedge \mu_2$ is also normalized.*

If μ is a normalized fuzzy filter, then μ^+ is also normalized.

Lemma 4.2. $X_\mu = X_{\mu^+}$.

Lemma 4.3. $(\Im(X), \leq)$ *is a meet-semi lattice. It has a smallest element* $\chi_{\{1\}}$ *and the largest element* $1C$ *given by* $1C(x) = 1\ \forall\ x \in X$.

If $\mu^+(x) = 0$ for some $x \in X$, then $\mu(x) = 0$.

Lemma 4.4. *If F_1 and F_2 are filters of X, then*

$$\chi_{F_1 \cap F_2} = \chi_{F_1} \wedge \chi_{F_2}.$$

If μ_1 and μ_2 are two normalized fuzzy filters of X, then

$$X_{\mu_1 \wedge \mu_2} = X_{\mu_1} \cap X_{\mu_2}.$$

Therefore,

$$\theta(F_1 \cap F_2) = \theta(F_1) \wedge \theta(F_2)$$

and

$$\gamma(\mu_1 \wedge \mu_2) = \gamma(\mu_1) \cap \gamma(\mu_2).$$

Definition 4.2. A fuzzy filter μ is a fuzzy ultrafilter if it is non constant and μ^+ is a maximal element of $(\Im(X), \leq)$.

Proposition 4.2. *If μ is non-constant and is a maximal element of $(\Im(X), \leq)$, then it takes only the values {0,1}.*

Proof. By hypothesis, μ is non constant and $\mu(1) = 1$. We claim that if $\mu(x) \neq 1$, then $\mu(x) = 0$. If not, there exists $a \in X$ such that $0 < \mu(a) < 1$. Let $\alpha(x) = 1/2\{1 + \mu(x)\}$, if $\mu(x) \geq 1/2$ and $\alpha(x) = 3/4\mu(x)$, if $\mu(x) < 1/2$.

One can observe that

$\alpha(x) \geq 3/4$ if and only if $\mu(x) \geq 1/2$; $\alpha(x) < 3/4$ if and only if $\mu(x) < 1/2$;

α is a fuzzy subset of X and $\alpha(1) = 1 \geq \alpha(x)$ for any $x \in X$ and $\alpha(x_0) = 0$ for any $x_0 \in X$ is such that $\mu(x_0) = 0$.

Now, let $t \in [0, 1]$.

If $t \geq 3/4$, then $\alpha_t = \{x/\alpha(x) \geq t\} = \mu_{1/2}$.

If $t < 3/4$, then

$$\begin{aligned} \alpha_t &= \{x/\alpha(x) \geq t\} && = \{x/\alpha(x) \geq 3/4\} \cup \{x/t \leq \alpha(x) < 3/4\} \\ &= \{x/\mu(x) \geq 2t/3\} && = \mu_{2t/3}. \end{aligned}$$

We obtain that for all $t \in [0.1]$, α_t is either empty or a filter of X, hence we can conclude that α is a normalized fuzzy filter. However $\alpha(x) \geq \mu(x)\ \forall\ x \in X$ and $\alpha(a) > \mu(a)$ and we have a contradiction since μ is maximal in the set of normalized fuzzy filters of X. □

Theorem 4.1. *Every fuzzy ultrafilter of X is normalized and takes only the values {0,1}.*

Proof. If μ is an ultrafilter, μ^+ is a maximal in the set of all normalized fuzzy filters of X. Since μ is non constant, μ^+ is also non constant. We use the Proposition 4.2 and obtain that μ^+ takes only the values {0,1}, Using Lemma 4.3 and the definition of μ^+, we can prove that $\mu^+ = \mu$. □

Corollary 4.1. *If μ is a fuzzy ultrafilter of X, then $\chi_{X_\mu} = \mu$.*

Corollary 4.2. *If μ is a fuzzy ultrafilter of X, then X_μ is an ultrafilter of X.*

Let us recall the following results:

Theorem 4.2. (Corollary 3.9. of [1]) *Every fuzzy prime ideal μ of X takes only two values $\{\mu(0), \mu(1)\}$.*

Theorem 4.3. (Theorem 3.9. of [3]) *Every ultrafilter of X is prime.*

Theorem 4.4. *Every fuzzy ultrafilter of X is fuzzy prime.*

Proof. By definition of fuzzy filter, we have

$$\mu(x \vee y) \geq \max(\mu(x), \mu(y)).$$

By Theorem 4.1, μ takes only the value {0,1}. To prove that $\mu(x \vee y) \leq \max(\mu(x), \mu(y))$, we need only to consider the case $\mu(x \vee y) = 1$. If $\mu(x \vee y) = 1$, then $x \vee y \in X_\mu$. From Corollary 4.2, X_μ is an ultrafilter of X, we apply Theorem 4.3 and obtain that X_μ is a prime filter. Therefore $x \in X_\mu$ or $y \in X_\mu$ and we have

$$\max(\mu(x), \mu(y)) = 1.$$

Finally $\mu(x \vee y) = \max(\mu(x), \mu(y))$ and μ is fuzzy prime. □

We can establish a correspondence between ultrafilter and fuzzy ultrafilter of X as follows: Let $\Im(X)'$ be the set of fuzzy ultrafilters of X and $F(X)'$ the set of ultrafilters of X. We define θ and γ as follows:

$$\theta : F(X) \longmapsto \Im(X) \text{ such that } \theta(F) = \chi_F$$
$$\gamma : \Im(X)' \longmapsto F(X)' \text{ such that } \gamma(\mu) = X_\mu$$

Proposition 4.3. *θ and γ are inverses of each other and we have a one-to-one correspondence between the ultrafilters and the fuzzy ultrafilters of X.*

Sketch of Proof. We can easily establish that for any ultrafilter F of X, $\gamma\theta(F) = F$ and for any fuzzy ultrafilter μ of $\Im(X)'$,

$$\theta\gamma(\mu) = \chi_{X_\mu} = \mu.$$

We can show that fuzzy filters and fuzzy ideals of X come in pairs. We recall that for any fuzzy subset μ of X, we can define a complement $\bar{\mu}$ of μ by:

$$\bar{\mu}(x) = 1 - \mu(x).$$

□

Proposition 4.4. *If a fuzzy subset μ of X is a fuzzy ideal, then $\mu(x) \geq \mu(y)$ when $x \leq y$.*

Proof. Since $x \leq y$, $x * y = 0$. From the definition of fuzzy ideal, we obtain that

$$\mu(x) \geq \min(\mu(x * y), \mu(y)) = \mu(y).$$

□

Theorem 4.5. *Let μ be a fuzzy subset of X, if μ is a fuzzy prime ideal, then its complement $\bar{\mu}$ is a fuzzy filter, in fact, a fuzzy ultrafilter.*

Proof. Let $x, y \in X$ such that $x \leq y$. Since μ is a fuzzy ideal, we apply the Proposition 4.4 and obtain $\mu(x) \geq \mu(y)$. So $1-\mu(x) \leq 1-\mu(y)$ and $\bar{\mu}(x) \leq \bar{\mu}(y)$. On the other hand, since μ is a fuzzy prime ideal, $\mu(x \wedge y) = \max(\mu(x), \mu(y))$. Therefore,

$$1 - \mu(x \wedge y) = (1 - \max(\mu(x), \mu(y))) = \min(1 - \mu(x), 1 - \mu(y))$$

and we obtain

$$\bar{\mu}(x \wedge y) \geq \min(\bar{\mu}(x), \bar{\mu}(y)).$$

Thus, $\bar{\mu}$ is a fuzzy filter. Because μ is prime, μ takes only two values $\{0,1\}$, it is easy to see that $\bar{\mu}$ also takes only two values $\{0,1\}$. From Theorem 4.1, we conclude that $\bar{\mu}$ is a fuzzy ultrafilter. □

5 Conclusion and further Suggestions

We have established some properties of fuzzy filters introduced in [4] and constructed some algorithms for recognizing filters and fuzzy filters. In [7], J. Meng and X. Gou gave a procedure which generate a fuzzy ideal in a BCI-algebra. Since we have proved that fuzzy ideal and fuzzy filter come in pair, a natural question is to describe and find a procedure to construct the fuzzy filter generated by a fuzzy set.

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A Algorithms

Algorithm for BCI-algebras

```
Input(X : set, ∗: binary operation)
Output("X is a BCI-algebra or not")
Begin
 If X = ∅ then
   go to (1.);
 EndIf
 If 0 ∉ X then
   go to (1.);
 EndIf
 Stop:=false;
 i := 1;
 While i ≤ |X| and not(Stop) do
  If x_i ∗ x_i ≠ 0 then
   Stop:=true;
  EndIf
  j := 1
  While j ≤ |X| and not(Stop) do
   If x_i ∗ (x_i ∗ y_j) ≠ 0 then
    Stop:=true;
   EndIf
   If (x_i ∗ y_j = 0) and (y_j ∗ x_i = 0) then
    If x_i ≠ y_j then
     Stop:=true;
    EndIf
   EndIf
   k := 1;
   While k ≤ |X| and not(Stop) do
    If ((x_i ∗ y_j) ∗ (x_i ∗ z_k)) ∗ (z_k ∗ y_j) ≠ 0 then
     Stop:=true;
    EndIf
   EndWhile
  EndWhile
 EndWhile
 If Stop then
  (1.) Output("X is not a BCI-algebra")
 Else
  Output("X is a BCI-algebra")
 EndIf
End
```

Algorithm for filters of BCI-algebra

Input(X : *BCI-algebra*, $F \subset X$);
Output("F is a filter of X or not");
Begin
  If $F = \emptyset$ **then**
    go to (1.);
  EndIf
  Stop:=*false*;
  $i := 1$;
  While $i \leq |X|$ **and not**(*Stop*) **do**
    $j := 1$
    While $j \leq |X|$ **and not**(*Stop*) **do**
      If $x_i \in F$ **and** $x_i \leq y_j$ **then**
        If $y_j \notin F$ **then**
          Stop:=*true*;
        EndIf
      EndIf
      If not(*Stop*) **then**
        If $x_i \in F$ **and** $y_j \in F$ **then**
          If $(x_i \wedge y_j) \notin F$ **or** $(y_j \wedge x_i) \notin F$ **then**
            Stop:=*true*;
          EndIf
        EndIf
      EndIf
    EndWhile
  EndWhile
  If *Stop* **then**
    Output("F is not a filter of X")
  Else
    Output("F is a filter of X")
  EndIf
End

Algorithm for fuzzy subsets

```
Input(X : BCI-algebra, A : X ⟶ [0,1]);
Output("A is a fuzzy subset of X or not");
Begin
  Stop:=false;
  i := 1;
  While i ≤ |X| and not(Stop) do
    If (A(x_i) < 0) or (A(x_i) > 1) then
      Stop:=true;
    EndIf
  EndWhile
  If Stop then
    Output("A is a fuzzy subset of X")
  Else
    Output("A is not a fuzzy subset of X")
  EndIf
End
```

Algorithm for fuzzy filters

```
Input(X: Commutative BCI-algebra, μ: Fuzzy subset);
Output("μ is a fuzzy filter or not");
Begin
  Stop:=false;
  i := 1;
  While i ≤ |X| and not(Stop) do
     j := 1
    While j ≤ |X| and not(Stop) do
      If μ(x_i ∧ y_j) < min(μ(x_i), μ(y_j)) then
        Stop:=true;
      EndIf
      If not(Stop) then
        If y_j ≥ x_i then
          If μ(y_j) < μ(x_i) then
            Stop:=true;
          EndIf
        EndIf
      EndIf
    EndWhile
  EndWhile
  If Stop then
    Output("I is not a fuzzy filter")
  Else
    Output("I is a fuzzy filter")
  EndIf
End
```