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A NEW ALGORITHM FOR COMPUTING IMPLIED VOLATILITY

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Abstract

In this study, we simplified the Black-Scholes formula to a two-input version. This simplified formula presents a one-to-one relationship with one input given that the other input is fixed. With this simplified formula, we created an option-price data grid and showed that the implied volatility can be obtained by interpolation. This interpolation-based algorithm does not require iteration and has an adjustable accuracy, which is very useful in computing implied volatilities for a large number of options in a real-time environment.

1 Introduction

The classical Black-Scholes (BS) formula is the most commonly used formula for obtaining the fair price of European options in the arbitrage-free framework. In the recent years, European options have gained popularity among future exchanges as one of their main products. Other than using the BS formula to calculate the fair price, one major application of the BS formula is to extract the volatility that makes the fair price equal to the current market price of European options. This volatility is also known as the implied volatility.

The exact closed-form formula for the implied volatility has not yet been found and is likely to remain unfound. To the best of our knowledge, there are two main methods used to compute the implied volatility. The first method is

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to find the numerical solution using iterative search methods such as the Bisection method [1], the Newton-Raphson method, and the Dekker-Brent method. According to Li [2], the Dekker-Brent method is chosen by MATLAB to calculate the implied volatility based on the BS formula. The second method involves using a Taylor series expansion to obtain the approximate formula for the implied volatility. There are a number of formulas developed based on this method, such as the Brenner-Subrahmanyam formula [3], the Corrado and Miller formula [4], the Chance formula [5], and the Li formula [6]. Furthermore, other approximating formulas that are not based on a Taylor series expansion are also available, such as the Li formula [2].

The iterative search method can achieve the highest accuracy, but it may run into a time complexity problem caused by a large number of iterations. In contrast, the approximate formula is considered to be faster than the iterative search method. However, there exists an accuracy issue in the approximate formula.

In practice, there are many market participants using implied volatility for various purposes such as pricing options, selecting options, and calculating Greeks. Therefore, the need for using implied volatility is not only at the end of day. Professional investors and traders need to know an accurate implied volatility on a real-time basis for their decision making process. The real-time implied volatility can be used on a real-time basis to calculate other variables such as Delta, Gamma, effective gearing, and time decay. For this reason, accuracy is a preferred choice over speed. Otherwise, other variables that use the implied volatility as an input will not be accurate. For a large corporation, it is possible to calculate a real-time implied volatility using a high-performance platform, but some investors and traders may have only a personal computer and a basic spreadsheet to calculate the implied volatility, which is fine for a small number of options. However, in a case when there are a large number of options to be calculated, the performance of a small platform may not be sufficient to handle a large number of iterations concurrently.

To respond to the needs of professional investors and traders, our new algorithm aims to achieve both accuracy and computation speed. Our algorithm starts with grouping variables together to simplify the BS formula. With the simplified BS formula, we can create a two-dimension table. By using an interpolation technique, the table can easily lead to an implied volatility without using iterations. Moreover, the accuracy of this algorithm can be adjusted by changing the intervals of the data grid.

2 Simplified BS formula

The BS formula developed by Black-Scholes [7] and Merton [8] can be used to calculate the price of European call and put options. Under the BS framework, the value of a call option for a non-dividend-paying underlying asset is:

$$c = SN(d_1) - Ke^{-rT}N(d_2) \tag{1}$$

Based on the put-call parity, the value of a put option for a non-dividend-paying underlying asset is:

$$p = Ke^{-rT}N(-d_2) - SN(-d_1) \tag{2}$$

where

$$d_1 = \frac{log\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{3}$$

$$d_2 = \frac{log\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{4}$$

For all of the above equations, the cumulative normal distribution function, the spot price of the underlying asset, the strike price, the risk-free rate, and the time-to-expiration are denoted as $N(\cdot)$, S, K, r, T, and σ , respectively.

We begin the simplification by defining the time-uncertainty (U), the discount-moneyness (M), the call-to-spot ratio (c_s) the put-to-spot ratio (p_s) as follows:

$$U = \sigma\sqrt{T} \tag{5}$$

$$M = \left(\frac{S}{K}\right)e^{rT} \tag{6}$$

$$c_s = \frac{c}{S} \tag{7}$$

$$p_s = \frac{p}{S} \tag{8}$$

It is straightforward to show that by substituting (5), (6), (7), and (8) into (1), (2), (3), and (4), the simplified BS formulas for the call price and the put price are:

$$c_s = N\left(\frac{log(M)}{U} + \frac{U}{2}\right) - M^{-1}N\left(\frac{log(M)}{U} - \frac{U}{2}\right) \tag{9}$$

$$p_s = M^{-1}N\left(-\frac{log(M)}{U} + \frac{U}{2}\right) - N\left(-\frac{log(M)}{U} - \frac{U}{2}\right) \tag{10}$$

In fact, the time-uncertainty and the discount-moneyness are not independent because both values have the time-to-expiration variable as their common parameter. The effect of the time-to-expiration on the discount-moneyness is considered to be insignificant, which allows us to neglect this effect. Nonetheless, formulas (9) and (10) are basically the BS formulas for calculating a European option price in terms of a percentage of the spot price of the underlying asset, as shown in Figure1 and Figure 2.

Figure 1: Call-to-spot ratio computed by the simplified BS formula

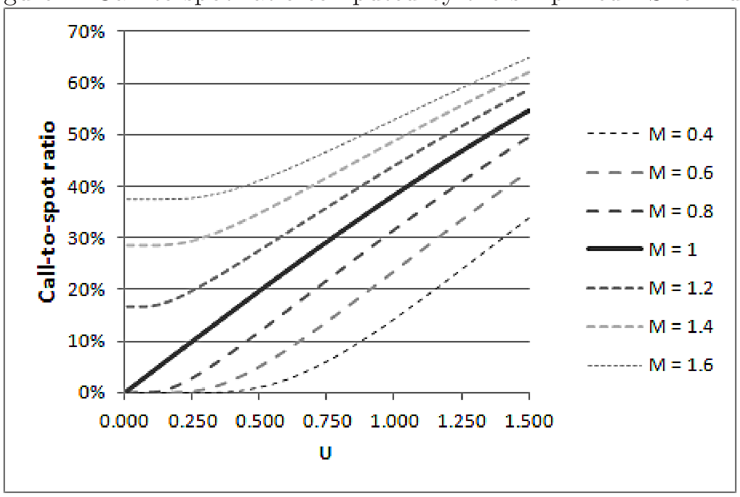
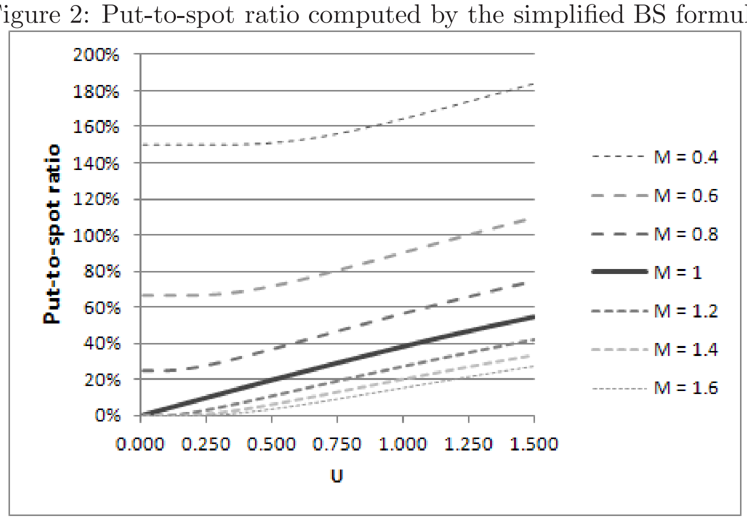


Figure 2: Put-to-spot ratio computed by the simplified BS formula



In Figure 1, it can be seen that the relationship between the call-to-spot ratio and the time-uncertainty is a one-to-one function for a fixed discount-moneyness. Likewise, the put-to-spot ratio in Figure 2 is a one-to-one function of the time-uncertainty for a fixed discount-moneyness. Consequently, the call-to-spot and the put-to-spot ratio will converge to their intrinsic values when the time-uncertainty approaches zero. The convergence will be even faster when the discount-moneyness deviates further from unity. In other words, when the moneyness of the option is more out-of-the-money or more in-the-money, the option price will converge to its intrinsic value more quickly. As shown in Figure 1, we can see clearly that there are almost no differences between the call-to-spot ratios with a discount-moneyness of 0.4 when the time-uncertainty is lower than 0.250. This is also true for the put-to-spot ratio. Unsurprisingly, the European option price based on the BS formula has the same effect when the time-to-maturity or the volatility is close to zero.

3 Numerical methods for computing implied volatility

Traditionally, the implied volatility can be extracted from the BS formula by assuming that all of the other variables are known. To compute the implied volatility numerically, we can employ any root-finding technique such as Bisection, Newton-Raphson, Secant, and Brent.

Because the simplified BS formula is equivalent to the BS formula, we can use the same numerical methods that we used to compute the implied volatility to compute the implied time-uncertainty (Implied U) in the simplified BS formula. Then, we can divide the implied time-uncertainty by the square root of the time-to-expiration to obtain the implied volatility.

In this paper, we used the Bisection method to compute the implied volatility because of its simplicity. First, we assumed that the spot price of the underlying asset, the strike price, the risk-free rate, and the time-to-expiration are known variables. Furthermore, we defined an arbitrarily chosen volatility, namely the "actual volatility (Vol)". Using all of the variables, the European option price could be calculated by the BS formula. With the European option price and other variables, it was very straightforward to employ the Bisection method to obtain the implied volatility from the classical BS formula and the simplified BS formula. Therefore, we defined IV_1 and IV_2 as the implied volatility from the classical BS formula and the simplified BS formula, respectively. Surely, both of the implied volatilities are not error-free. We do need to specify the error tolerance (ϵ) for the root-finding technique as well.

We demonstrated the computing procedure as mentioned above through three examples of European call options, including at-the-money (spot price of underlying asset = strike price), out-of-the-money (spot price of underlying asset < strike price), and in-the-money (spot price of underlying asset > strike price). We made the out-of-the-money and the in-the-money cases more extreme by choosing the spot price of the underlying asset to be higher or lower than the strike price 40%. The examples describing the at-the-money, the out-of-the-money, and the in-the-money cases are presented in Table 1, Table 2, and Table 3, respectively. We have to note that the error tolerance (ϵ) is 0.000001 and that the only two variables that remained unchanged were the strike price and the risk-free rate. To keep the examples simple, we intentionally chose the risk-free rate to be zero.

Table 1: Implied volatilities computed by the Bisection method (at-the-money)

Assumption : S = 100, X = 100, r = 0, actual volatility (Vol) = 60%

BS Formula			Simplified BS Formula				Absolute Error		
T	c	IV_1	M	c_s	Implied U	IV_2	$\lvert Vol-IV_1 \rvert$	$\lvert Vol-IV_2 \rvert$	$\lvert IV_1-IV_2 \rvert$
0.100	7.5581	60.0000%	1.0	0.075581	18.9737%	60.0000%	0.0000%	0.0000%	0.0000%
0.090	7.1713	60.0000%	1.0	0.071713	18.0000%	60.0000%	0.0000%	0.0000%	0.0000%
0.080	6.7622	60.0000%	1.0	0.067622	16.9706%	60.0000%	0.0000%	0.0000%	0.0000%
0.070	6.3264	60.0000%	1.0	0.063264	15.8745%	60.0000%	0.0000%	0.0000%	0.0000%
0.060	5.8580	60.0000%	1.0	0.058580	14.6969%	60.0000%	0.0000%	0.0000%	0.0000%
0.050	5.3484	60.0000%	1.0	0.053484	13.4164%	60.0000%	0.0000%	0.0000%	0.0000%
0.040	4.7844	60.0000%	1.0	0.047844	12.0000%	60.0000%	0.0000%	0.0000%	0.0000%
0.030	4.1441	60.0000%	1.0	0.041441	10.3923%	60.0000%	0.0000%	0.0000%	0.0000%
0.020	3.3841	60.0000%	1.0	0.033841	8.4853%	60.0000%	0.0000%	0.0000%	0.0000%
0.018	3.2106	60.0000%	1.0	0.032106	8.0498%	60.0000%	0.0000%	0.0000%	0.0000%
0.016	3.0270	60.0000%	1.0	0.030270	7.5895%	60.0000%	0.0000%	0.0000%	0.0000%
0.014	2.8316	60.0000%	1.0	0.028316	7.0993%	60.0000%	0.0000%	0.0000%	0.0000%
0.012	2.6216	60.0000%	1.0	0.026216	6.5727%	60.0000%	0.0000%	0.0000%	0.0000%
0.010	2.3933	60.0000%	1.0	0.023933	6.0000%	60.0000%	0.0000%	0.0000%	0.0000%
0.008	2.1407	60.0000%	1.0	0.021407	5.3666%	60.0000%	0.0000%	0.0000%	0.0000%
0.006	1.8539	60.0000%	1.0	0.018539	4.6476%	60.0000%	0.0000%	0.0000%	0.0000%
0.004	1.5138	60.0000%	1.0	0.015138	3.7947%	60.0000%	0.0000%	0.0000%	0.0000%
0.002	1.0704	60.0000%	1.0	0.010704	2.6833%	60.0000%	0.0000%	0.0000%	0.0000%
				Maximum Absolute Error			0.0000%	0.0000%	0.0000%

From the three tables 1, 2 and 3, we can see that the absolute errors of the implied volatilities are very low for the at-the-money and the out-of-the-money cases. However, the absolute errors are clearly evident for in-the-money case, especially when the time-to-expiration is lower than 0.006. In Table 3 (time-to-expiration = 0.002), the implied volatilities from the classical BS formula and the simplified BS formula are significantly different from the actual volatility. Even without using any approximating formula to compute the implied volatility, it is unavoidable to encounter a significant error.

Table 2: Implied volatilities computed by the Bisection method (out-of-the-money)

Assumption : S = 60, X = 100, r = 0, actual volatility (Vol) = 60%

BS Formula			Simplified BS Formula				Absolute Error		
T	c	IV_1	M	c_s	Implied U	IV_2	$\|Vol-IV_1\|$	$\|Vol-IV_2\|$	$\|IV_1-IV_2\|$
0.100	0.0159	60.0000%	0.6	0.000265	18.9737%	60.0000%	0.0000%	0.0000%	0.0000%
0.090	0.0093	60.0000%	0.6	0.000155	18.0000%	60.0000%	0.0000%	0.0000%	0.0000%
0.080	0.0048	60.0000%	0.6	0.000081	16.9706%	60.0000%	0.0000%	0.0000%	0.0000%
0.070	0.0021	60.0000%	0.6	0.000035	15.8745%	60.0000%	0.0000%	0.0000%	0.0000%
0.060	0.0007	60.0000%	0.6	0.000012	14.6969%	60.0000%	0.0000%	0.0000%	0.0000%
0.050	0.0002	60.0000%	0.6	0.000003	13.4164%	60.0000%	0.0000%	0.0000%	0.0000%
0.040	0.0000	60.0000%	0.6	0.000000	12.0000%	60.0000%	0.0000%	0.0000%	0.0000%
0.030	0.0000	60.0002%	0.6	0.000000	10.3923%	60.0002%	0.0002%	0.0002%	0.0000%
0.020	0.0000	60.0000%	0.6	0.000000	8.4853%	60.0000%	0.0000%	0.0000%	0.0000%
0.018	0.0000	60.0000%	0.6	0.000000	8.0498%	60.0000%	0.0000%	0.0000%	0.0000%
0.016	0.0000	60.0000%	0.6	0.000000	7.5895%	60.0000%	0.0000%	0.0000%	0.0000%
0.014	0.0000	60.0000%	0.6	0.000000	7.0993%	60.0000%	0.0000%	0.0000%	0.0000%
0.012	0.0000	60.0000%	0.6	0.000000	6.5727%	60.0000%	0.0000%	0.0000%	0.0000%
0.010	0.0000	60.0000%	0.6	0.000000	6.0000%	60.0000%	0.0000%	0.0000%	0.0000%
0.008	0.0000	60.0000%	0.6	0.000000	5.3666%	60.0000%	0.0000%	0.0000%	0.0000%
0.006	0.0000	60.0000%	0.6	0.000000	4.6476%	60.0000%	0.0000%	0.0000%	0.0000%
0.004	0.0000	60.0000%	0.6	0.000000	3.7947%	60.0000%	0.0000%	0.0000%	0.0000%
0.002	0.0000	60.0000%	0.6	0.000000	2.6833%	60.0000%	0.0000%	0.0000%	0.0000%
					Maximum Absolute Error		0.0002%	0.0002%	0.0000%

Table 3: Implied volatilities computed by the Bisection method (in-the-money)

Assumption : S = 140, X = 100, r = 0, actual volatility (Vol) = 60%

BS Formula			Simplified BS Formula				Absolute Error		
T	c	IV_1	M	c_s	Implied U	IV_2	$\|Vol-IV_1\|$	$\|Vol-IV_2\|$	$\|IV_1-IV_2\|$
0.100	40.3414	60.0000%	1.4	0.288153	18.9737%	60.0000%	0.0000%	0.0000%	0.0000%
0.090	40.2541	60.0000%	1.4	0.287529	18.0000%	60.0000%	0.0000%	0.0000%	0.0000%
0.080	40.1781	60.0000%	1.4	0.286986	16.9706%	60.0000%	0.0000%	0.0000%	0.0000%
0.070	40.1148	60.0000%	1.4	0.286534	15.8745%	60.0000%	0.0000%	0.0000%	0.0000%
0.060	40.0655	60.0000%	1.4	0.286182	14.6969%	60.0000%	0.0000%	0.0000%	0.0000%
0.050	40.0310	60.0000%	1.4	0.285936	13.4164%	60.0000%	0.0000%	0.0000%	0.0000%
0.040	40.0106	60.0000%	1.4	0.285790	12.0000%	60.0000%	0.0000%	0.0000%	0.0000%
0.030	40.0020	60.0000%	1.4	0.285728	10.3923%	60.0000%	0.0000%	0.0000%	0.0000%
0.020	40.0001	60.0000%	1.4	0.285715	8.4853%	60.0000%	0.0000%	0.0000%	0.0000%
0.018	40.0000	60.0000%	1.4	0.285715	8.0498%	60.0000%	0.0000%	0.0000%	0.0000%
0.016	40.0000	60.0000%	1.4	0.285714	7.5895%	60.0000%	0.0000%	0.0000%	0.0000%
0.014	40.0000	60.0000%	1.4	0.285714	7.0993%	60.0000%	0.0000%	0.0000%	0.0000%
0.012	40.0000	60.0000%	1.4	0.285714	6.5727%	60.0000%	0.0000%	0.0000%	0.0000%
0.010	40.0000	60.0000%	1.4	0.285714	6.0000%	60.0000%	0.0000%	0.0000%	0.0000%
0.008	40.0000	59.9999%	1.4	0.285714	5.3666%	59.9999%	0.0001%	0.0001%	0.0000%
0.006	40.0000	60.1243%	1.4	0.285714	4.6605%	60.1669%	0.1243%	0.1669%	0.0426%
0.004	40.0000	70.3325%	1.4	0.285714	4.3857%	69.3448%	10.3325%	9.3448%	0.9878%
0.002	40.0000	97.0939%	1.4	0.285714	4.3857%	98.0683%	37.0939%	38.0683%	0.9744%
					Maximum Absolute Error		37.0939%	38.0683%	0.9878%

4 New algorithm based on simplified BS formula

Compared to the classical BS formula, the simplified BS formula reduces the number of inputs from six variables to two variables. To compute the call-to-spot ratio or the put-to-spot ratio, the simplified BS formula requires only discount-moneyness and time-uncertainty, as shown in Table 4 and Table 5. We can obtain the implied time-uncertainty from Table 4 and Table 5 given that the discount-moneyness and the call-to-spot (or put-to-spot) ratio are known.

For example, one can obtain the implied time-uncertainty given that the call-to-spot ratio and the discount moneyness are 0.01 and 1, respectively. By interpolating the data in Table 4, it is very straightforward to show that the implied time-uncertainty is equal to 0.02506702. Providing that M = 1 and the call-to-spot ratio = 0.01, the implied time-uncertainty computed by the Bisection method is equal to 0.02506694. The absolute error of the implied time-uncertainty from our interpolation method and the Bisection method is approximately 0.00000008. Assuming that the time-to-expiration is 0.0001 (0.0365 calendar days), the absolute error of the implied volatility from a linear interpolation will be approximately 0.000008, or 0.0008

As mentioned earlier, the implied volatility calculated by the Bisection method may have a large error when it is close to maturity. The interpolated implied volatility is also expected to introduce a similar error. To check the accuracy of the linearly interpolated implied volatility, we created Tables 6-8 by comparing the implied volatility based on the Bisection method of the BS formula (IV_1) with the interpolated implied volatility based on the Bisection method of the simplified BS formula (IV_3). In this case, the interval of U in the data grid is 0.00001

As presented in Tables 6-8, the interpolated implied volatility generally has a greater absolute error compared to that from the Bisection method, but this is not true for the last two scenarios in Table 8 (in-the-money). When the time-to-expiration is less than 0.006, the absolute error of the interpolated implied volatility is obviously lower than for any other method. In general, the absolute errors of the interpolated implied volatility in Tables 6-8 are lower than 0.01%, which is practically useful.

5 Conclusion

With the classical BS formula, the implied volatility of the European options can be easily obtained using iterative search methods. However, computing the implied volatility with iterative search methods can present a time complexity when it comes to a real-time calculation for a large number of options. In addition to iterative search methods, the approximating formulas for the implied volatility are also available, but the accuracy of these approximating formulas is not widely accepted among professional investors and traders. For this reason, we are interested in finding an algorithm that can quickly and accurately compute the implied

Table 4: Call-to-spot ratio computed by the simplified BS formula

	Discount-moneyness (M)				
Time-uncertainty (U)	0.60	0.80	1.00	1.20	1.40
0.000010	0.000000	0.000000	0.000004	0.166667	0.285714
0.000100	0.000000	0.000000	0.000040	0.166667	0.285714
0.001000	0.000000	0.000000	0.000399	0.166667	0.285714
0.002000	0.000000	0.000000	0.000798	0.166667	0.285714
0.003000	0.000000	0.000000	0.001197	0.166667	0.285714
0.004000	0.000000	0.000000	0.001596	0.166667	0.285714
0.005000	0.000000	0.000000	0.001995	0.166667	0.285714
0.006000	0.000000	0.000000	0.002394	0.166667	0.285714
0.007000	0.000000	0.000000	0.002793	0.166667	0.285714
0.008000	0.000000	0.000000	0.003192	0.166667	0.285714
0.009000	0.000000	0.000000	0.003590	0.166667	0.285714
0.010000	0.000000	0.000000	0.003989	0.166667	0.285714
0.020000	0.000000	0.000000	0.007979	0.166667	0.285714
0.030000	0.000000	0.000000	0.011968	0.166667	0.285714
0.040000	0.000000	0.000000	0.015957	0.166667	0.285714
0.050000	0.000000	0.000000	0.019945	0.166668	0.285714
0.100000	0.000000	0.000499	0.039878	0.167894	0.285723
0.150000	0.000016	0.005045	0.059785	0.174094	0.286261
0.200000	0.000435	0.014824	0.079656	0.184561	0.288929
0.250000	0.002423	0.028320	0.099476	0.197549	0.294385
0.300000	0.006976	0.044180	0.119235	0.212005	0.302260

Table 5: Put-to-spot ratio computed by the simplified BS formula

	Discount-moneyness (M)				
Time-uncertainty (U)	0.60	0.80	1.00	1.20	1.40
0.000010	0.666667	0.250000	0.000004	0.000000	0.000000
0.000100	0.666667	0.250000	0.000040	0.000000	0.000000
0.001000	0.666667	0.250000	0.000399	0.000000	0.000000
0.002000	0.666667	0.250000	0.000798	0.000000	0.000000
0.003000	0.666667	0.250000	0.001197	0.000000	0.000000
0.004000	0.666667	0.250000	0.001596	0.000000	0.000000
0.005000	0.666667	0.250000	0.001995	0.000000	0.000000
0.006000	0.666667	0.250000	0.002394	0.000000	0.000000
0.007000	0.666667	0.250000	0.002793	0.000000	0.000000
0.008000	0.666667	0.250000	0.003192	0.000000	0.000000
0.009000	0.666667	0.250000	0.003590	0.000000	0.000000
0.010000	0.666667	0.250000	0.003989	0.000000	0.000000
0.020000	0.666667	0.250000	0.007979	0.000000	0.000000
0.030000	0.666667	0.250000	0.011968	0.000000	0.000000
0.040000	0.666667	0.250000	0.015957	0.000000	0.000000
0.050000	0.666667	0.250000	0.019945	0.000001	0.000000
0.100000	0.666667	0.250499	0.039878	0.001228	0.000008
0.150000	0.666683	0.255045	0.059785	0.007427	0.000546
0.200000	0.667102	0.264824	0.079656	0.017894	0.003215
0.250000	0.669090	0.278320	0.099476	0.030882	0.008671
0.300000	0.673643	0.294180	0.119235	0.045338	0.016546

Table 6: Implied volatilities computed by the Bisection method and the interpolation method (at-the-money)
Assumption : S = 100, X = 100, r = 0, actual volatility (Vol) = 60%

BS Formula			Simplified BS Formula				Absolute Error		
T	c	IV_1	M	c_s	Implied U	IV_3	$\|Vol-IV_1\|$	$\|Vol-IV_3\|$	$\|IV_1-IV_3\|$
0.100	7.5581	60.0000%	1.0	0.075581	18.9747%	60.0032%	0.0000%	0.0032%	0.0032%
0.090	7.1713	60.0000%	1.0	0.071713	18.0010%	60.0033%	0.0000%	0.0033%	0.0033%
0.080	6.7622	60.0000%	1.0	0.067622	16.9716%	60.0035%	0.0000%	0.0035%	0.0035%
0.070	6.3264	60.0000%	1.0	0.063264	15.8755%	60.0038%	0.0000%	0.0038%	0.0038%
0.060	5.8580	60.0000%	1.0	0.058580	14.6979%	60.0041%	0.0000%	0.0041%	0.0041%
0.050	5.3484	60.0000%	1.0	0.053484	13.4174%	60.0045%	0.0000%	0.0045%	0.0045%
0.040	4.7844	60.0000%	1.0	0.047844	12.0010%	60.0050%	0.0000%	0.0050%	0.0050%
0.030	4.1441	60.0000%	1.0	0.041441	10.3933%	60.0058%	0.0000%	0.0058%	0.0058%
0.020	3.3841	60.0000%	1.0	0.033841	8.4863%	60.0071%	0.0000%	0.0071%	0.0071%
0.018	3.2106	60.0000%	1.0	0.032106	8.0508%	60.0075%	0.0000%	0.0075%	0.0075%
0.016	3.0270	60.0000%	1.0	0.030270	7.5905%	60.0079%	0.0000%	0.0079%	0.0079%
0.014	2.8316	60.0000%	1.0	0.028316	7.1003%	60.0085%	0.0000%	0.0085%	0.0085%
0.012	2.6216	60.0000%	1.0	0.026216	6.5737%	60.0091%	0.0000%	0.0091%	0.0091%
0.010	2.3933	60.0000%	1.0	0.023933	6.0010%	60.0100%	0.0000%	0.0100%	0.0100%
0.008	2.1407	60.0000%	1.0	0.021407	5.3676%	60.0112%	0.0000%	0.0112%	0.0112%
0.006	1.8539	60.0000%	1.0	0.018539	4.6486%	60.0129%	0.0000%	0.0129%	0.0129%
0.004	1.5138	60.0000%	1.0	0.015138	3.7957%	60.0158%	0.0000%	0.0158%	0.0158%
0.002	1.0704	60.0000%	1.0	0.010704	2.6843%	60.0224%	0.0000%	0.0224%	0.0224%
Maximum Absolute Error							0.0000%	0.0224%	0.0224%

Table 7: Implied volatilities computed by the Bisection method and the interpolation method (out-of-the-money)
Assumption : S = 60, X = 100, r = 0, actual volatility (Vol) = 60%

BS Formula			Simplified BS Formula				Absolute Error		
T	c	IV_1	M	c_s	Implied U	IV_3	$\|Vol-IV_1\|$	$\|Vol-IV_3\|$	$\|IV_1-IV_3\|$
0.100	0.0159	60.0000%	0.6	0.000265	18.9747%	60.0032%	0.0000%	0.0032%	0.0032%
0.090	0.0093	60.0000%	0.6	0.000155	18.0010%	60.0033%	0.0000%	0.0033%	0.0033%
0.080	0.0048	60.0000%	0.6	0.000081	16.9716%	60.0035%	0.0000%	0.0035%	0.0035%
0.070	0.0021	60.0000%	0.6	0.000035	15.8755%	60.0038%	0.0000%	0.0038%	0.0038%
0.060	0.0007	60.0000%	0.6	0.000012	14.6979%	60.0041%	0.0000%	0.0041%	0.0041%
0.050	0.0002	60.0000%	0.6	0.000003	13.4174%	60.0045%	0.0000%	0.0045%	0.0045%
0.040	0.0000	60.0000%	0.6	0.000000	12.0010%	60.0050%	0.0000%	0.0050%	0.0050%
0.030	0.0000	60.0002%	0.6	0.000000	10.3933%	60.0059%	0.0002%	0.0059%	0.0057%
0.020	0.0000	60.0000%	0.6	0.000000	8.4863%	60.0071%	0.0000%	0.0071%	0.0071%
0.018	0.0000	60.0000%	0.6	0.000000	8.0508%	60.0075%	0.0000%	0.0075%	0.0075%
0.016	0.0000	60.0000%	0.6	0.000000	7.5905%	60.0079%	0.0000%	0.0079%	0.0079%
0.014	0.0000	60.0000%	0.6	0.000000	7.1003%	60.0084%	0.0000%	0.0084%	0.0084%

Table 7 contd.: Implied volatilities computed by the Bisection method and the interpolation method (out-of-the-money) Assumption : S = 60, X = 100, r = 0, actual volatility (Vol) = 60%

BS Formula			Simplified BS Formula				Absolute Error		
T	c	IV_1	M	c_s	Implied U	IV_3	$\|Vol-IV_1\|$	$\|Vol-IV_3\|$	$\|IV_1-IV_3\|$
0.012	0.0000	60.0000%	0.6	0.000000	6.5737%	60.0091%	0.0000%	0.0091%	0.0091%
0.010	0.0000	60.0000%	0.6	0.000000	6.0010%	60.0100%	0.0000%	0.0100%	0.0100%
0.008	0.0000	60.0000%	0.6	0.000000	5.3676%	60.0112%	0.0000%	0.0112%	0.0112%
0.006	0.0000	60.0000%	0.6	0.000000	4.6486%	60.0129%	0.0000%	0.0129%	0.0129%
0.004	0.0000	60.0000%	0.6	0.000000	3.7957%	60.0157%	0.0000%	0.0157%	0.0157%
0.002	0.0000	60.0000%	0.6	0.000000	2.6843%	60.0221%	0.0000%	0.0221%	0.0221%
Maximum Absolute Error							0.0002%	0.0221%	0.0221%

Table 8: Implied volatilities computed by the Bisection method and the interpolation method (in-the-money)
Assumption : S = 140, X = 100, r = 0, actual volatility (Vol) = 60%

BS Formula			Simplified BS Formula				Absolute Error		
T	c	IV_1	M	c_s	Implied U	IV_3	$\|Vol-IV_1\|$	$\|Vol-IV_3\|$	$\|IV_1-IV_3\|$
0.100	40.3414	60.0000%	1.4	0.288153	18.9747%	60.0032%	0.0000%	0.0032%	0.0032%
0.090	40.2541	60.0000%	1.4	0.287529	18.0010%	60.0033%	0.0000%	0.0033%	0.0033%
0.080	40.1781	60.0000%	1.4	0.286986	16.9716%	60.0035%	0.0000%	0.0035%	0.0035%
0.070	40.1148	60.0000%	1.4	0.286534	15.8755%	60.0038%	0.0000%	0.0038%	0.0038%
0.060	40.0655	60.0000%	1.4	0.286182	14.6979%	60.0041%	0.0000%	0.0041%	0.0041%
0.050	40.0310	60.0000%	1.4	0.285936	13.4174%	60.0045%	0.0000%	0.0045%	0.0045%
0.040	40.0106	60.0000%	1.4	0.285790	12.0010%	60.0050%	0.0000%	0.0050%	0.0050%
0.030	40.0020	60.0000%	1.4	0.285728	10.3933%	60.0058%	0.0000%	0.0058%	0.0058%
0.020	40.0001	60.0000%	1.4	0.285715	8.4863%	60.0071%	0.0000%	0.0071%	0.0071%
0.018	40.0000	60.0000%	1.4	0.285715	8.0508%	60.0075%	0.0000%	0.0075%	0.0075%
0.016	40.0000	60.0000%	1.4	0.285714	7.5905%	60.0079%	0.0000%	0.0079%	0.0079%
0.014	40.0000	60.0000%	1.4	0.285714	7.1003%	60.0085%	0.0000%	0.0085%	0.0085%
0.012	40.0000	60.0000%	1.4	0.285714	6.5737%	60.0091%	0.0000%	0.0091%	0.0091%
0.010	40.0000	60.0000%	1.4	0.285714	6.0010%	60.0100%	0.0000%	0.0100%	0.0100%
0.008	40.0000	59.9999%	1.4	0.285714	5.3676%	60.0112%	0.0001%	0.0112%	0.0112%
0.006	40.0000	60.1243%	1.4	0.285714	4.6570%	60.1216%	0.1243%	0.1216%	0.0027%
0.004	40.0000	70.3325%	1.4	0.285714	4.0480%	64.0045%	10.3325%	4.0045%	6.3280%
0.002	40.0000	97.0939%	1.4	0.285714	4.0480%	90.5160%	37.0939%	30.5160%	6.5778%
Maximum Absolute Error							37.0939%	30.5160%	6.5778%

volatility for a large number of options on a real-time basis to accommodate the needs of professional investors and traders.

The new algorithm presented in this paper starts by simplifying the BS formula to a two-input version of the BS formula, called the simplified BS formula. With the simplified BS formula, the option-price data grid can be generated based on only two variables, which are the discount-moneyness and the time-uncertainty. Then, we showed that the implied volatility could be extracted from the option-price data grid generated from the simplified BS formula by a linear interpolation. With the time-uncertainty interval of 0.00001, the absolute errors in most cases are considered to be acceptable for professional investors and traders. Relating to the speed of computation, creating an option-price data grid is definitely a time-consuming procedure in this algorithm. However, the option-price data grid needs to be generated only once at the beginning; for an ongoing process, we are not required to regenerate the option-price data grid. The interpolation process, which is the only ongoing process, requires no iterations at all. As a result, the speed of interpolating the implied volatility should be much faster than the iterative search methods, especially for a large real-time system.

In this paper, we intentionally used a linear interpolation to calculate the implied volatility to simplify the examples. For an industrial application, the interpolation techniques used in this algorithm must be able to handle a two-dimensional regular grid, such as a bilinear interpolation [9] or a bicubic interpolation [9]. Moreover, the accuracy of the interpolated implied volatility could be further improved by reducing the time-uncertainty interval and the discount-moneyness interval used in the option-price data grid.

With this algorithm, the implied volatility can be computed quickly, while the accuracy can be adjusted to meet the requirements of sophisticated investors. The advantage of this algorithm is amplified when there are many options to be calculated on a real-time basis. However, there are cases in which it is better to stick with an implied volatility calculated from the classical BS formula, such as for a small number of options or for a low-frequency calculation.

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